

FORM 3IBP



The College Board  
Advanced Placement Examination  
CALCULUS AB  
SECTION II

This insert may be used for reference and/or scratchwork as you answer the free-response questions, but be sure to show all your work on problems and write your answers in the pink booklet. No credit will be given for work shown on this green insert.

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CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination  $\ln x$  denotes the natural logarithm of  $x$  (that is, logarithm to the base  $e$ ). (2) Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

1. Let  $f$  be the function defined by  $f(x) = 7 - 15x + 9x^2 - x^3$  for all real numbers  $x$ .

- (a) Find the zeros of  $f$ .
- (b) Write an equation of the line tangent to the graph of  $f$  at  $x = 2$ .
- (c) Find the  $x$ -coordinates of all points of inflection of  $f$ . Justify your answer.

2. Let  $f$  be the function given by  $f(x) = \frac{9x^2 - 36}{x^2 - 9}$ .

- (a) Describe the symmetry of the graph of  $f$ .
- (b) Write an equation for each vertical and each horizontal asymptote of  $f$ .
- (c) Find the intervals on which  $f$  is increasing.
- (d) Using the results found in parts (a), (b), and (c), sketch the graph of  $f$  on the axes below.

3. A particle moves along the  $x$ -axis so that at any time  $t \geq 1$  its acceleration is given by  $a(t) = \frac{1}{t}$ . At time  $t = 1$ , the velocity of the particle is  $v(1) = -2$  and its position is  $x(1) = 4$ .

- (a) Find the velocity  $v(t)$  for  $t \geq 1$ .
- (b) Find the position  $x(t)$  for  $t \geq 1$ .
- (c) What is the position of the particle when it is farthest to the left?

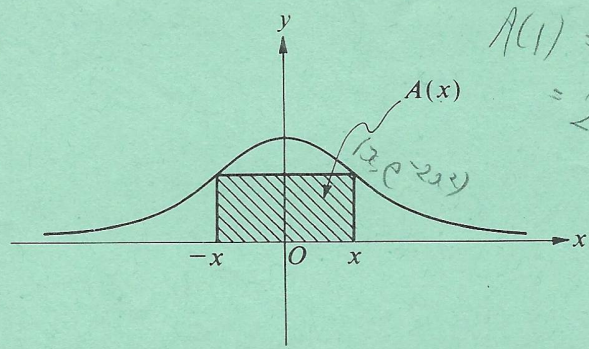
4. Let  $f$  be the function defined as follows.

$$f(x) = \begin{cases} |x - 1| + 2, & \text{for } x < 1 \\ ax^2 + bx, & \text{for } x \geq 1, \text{ where } a \text{ and } b \text{ are constants.} \end{cases}$$

- (a) If  $a = 2$  and  $b = 3$ , is  $f$  continuous for all  $x$ ? Justify your answer.
- (b) Describe all values of  $a$  and  $b$  for which  $f$  is a continuous function.
- (c) For what values of  $a$  and  $b$  is  $f$  both continuous and differentiable?

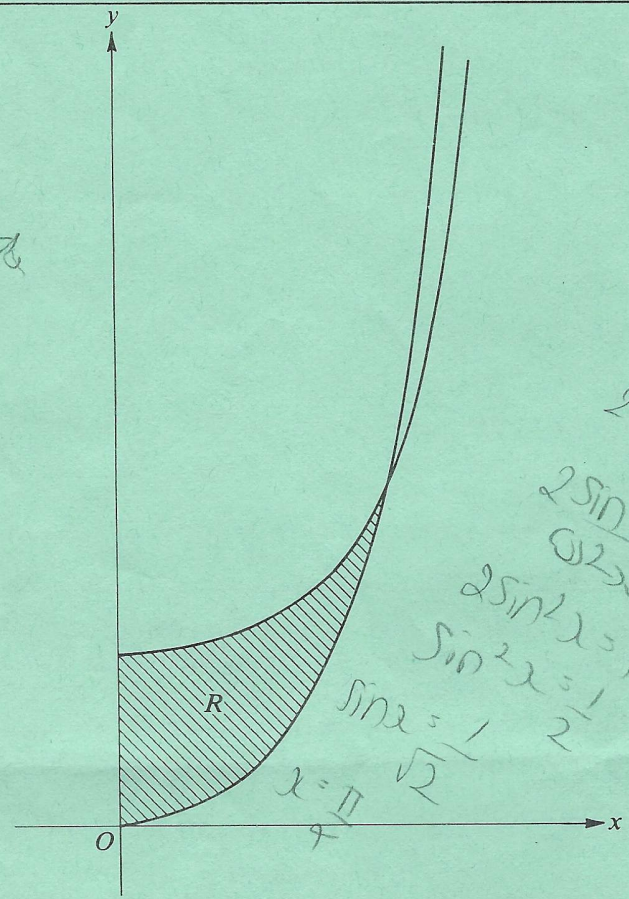
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$A(x) = 2x(e^{-2x^2})$   
 $A(1) = 2e^{-2(1)}$   
 $= 2e^{-2}$

5. Let  $A(x)$  be the area of the rectangle inscribed under the curve  $y = e^{-2x^2}$  with vertices at  $(-x, 0)$  and  $(x, 0)$ ,  $x \geq 0$ , as shown in the figure above.
- Find  $A(1)$ .  $2e^{-2}$
  - What is the greatest value of  $A(x)$ ? Justify your answer.  $A'(x) = 0$
  - What is the average value of  $A(x)$  on the interval  $0 \leq x \leq 2$ ?  $\frac{1}{2} \int_0^2 A(x) dx$



$\int_0^{\pi/4} \tan^2 x dx$

$\tan^2 x = \frac{1}{\cos^2 x}$   
 $2 \tan^2 x = \frac{1}{\cos^2 x}$   
 $\frac{2 \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$   
 $2 \sin^2 x = 1$   
 $\sin^2 x = \frac{1}{2}$   
 $\sin x = \frac{1}{\sqrt{2}}$   
 $x = \frac{\pi}{4}$

Which graph on top?

6. The shaded region  $R$  shown in the figure above is enclosed by the graphs of  $y = \tan^2 x$ ,  $y = \frac{1}{2} \sec^2 x$ , and the  $y$ -axis.
- Find the area of region  $R$ .  $R = \int_0^{\pi/4}$
  - Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid formed by revolving region  $R$  about the  $x$ -axis.

END OF EXAMINATION



$$7 - 747$$

$$x.p. | \int - 747$$

$$7 = \wedge x.p. \frac{7}{T} \quad \text{DP}$$

$$1 \neq \wedge p \quad 747 = n$$

n